The Calculus Logo

The above logo shows graphs, for $0 \le x \le \frac{\pi}{2}$, of certain functions in the family of functions

$$f_p(x) = \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)}, p \ge 0.$$

The substitution $t = \frac{\pi}{2} - x$ shows that

$$\int_0^{\pi/2} \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)} dx = \int_0^{\pi/2} \frac{\cos^p(x)}{\sin^p(x) + \cos^p(x)} dx.$$

Hence

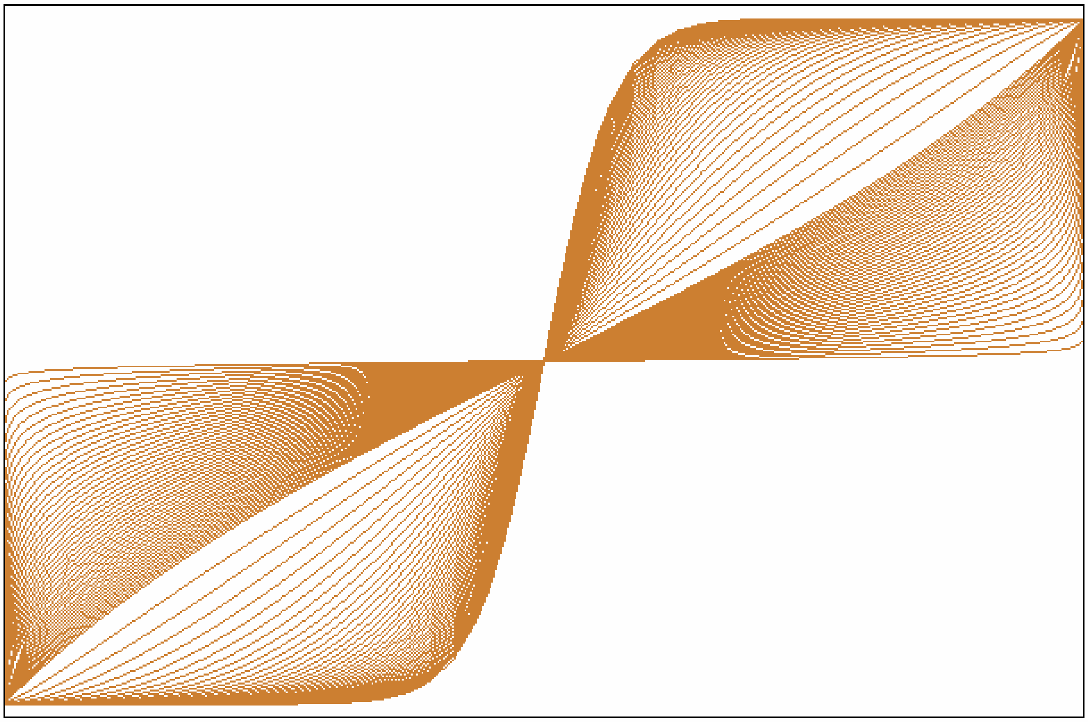
$$2\int_0^{\pi/2} \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)} dx = \int_0^{\pi/2} \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)} dx + \int_0^{\pi/2} \frac{\cos^p(x)}{\sin^p(x) + \cos^p(x)} dx$$

$$= \int_0^{\pi/2} dx = \frac{\pi}{2}$$

One gets $\int_0^{\pi/2} \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)} dx = \frac{\pi}{4}$. We conclude that the graph of each function f_p divides the rectangle $\left[0, \frac{\pi}{2}\right] \times [0,1]$ into two parts of the same area.

The graph of the function f_0 is the horizontal line $y = \frac{1}{2}$.

Parts of the graphs of the functions $f_p(x)$ approach the vertical line $x = \frac{\pi}{4}$ in the rectangle $\left[0, \frac{\pi}{2}\right] \times [0,1]$ as $p \to \infty$.



Graphs of functions $f_p(x) = \frac{\sin^p(x)}{\sin^p(x) + \cos^p(x)}$.